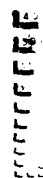


SPACE-TIME MODELLING WITH LONG-MEMORY DEPENDENCE:
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Space-time Modelling with Long-memory Dependence: Assessing Ireland's Wind Power Resource

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ABSTRACT

We consider estimation of the long-term average power output from a wind turbine generator at a site for which little data on wind speeds is available. Long-term records of wind speeds at the twelve synoptic meteorological stations are also used. Inference is based on a simple and parsimonious approximating model which accounts for the main features of wind speeds in Ireland, namely seasonal effects, spatial correlation, short-memory temporal autocorrelation, and long-memory temporal dependence. It synthesises deseasonalisation, kriging, ARMA modelling, and fractional differencing in a natural way. A simple kriging estimator performs well as a point estimator, and good interval estimators result from the model. The resulting procedure is easy to apply in practice.

Keywords: Deseasonalisation; Fractional differencing; Kriging; Optimal interpolation; Persistence.

1. INTRODUCTION

The Irish government has, in recent years, been considering the possibility of using wind energy to meet a significant portion of Ireland's energy needs. This paper describes a project aimed at developing methods for the evaluation of Ireland's wind power resource.

This resource may be exploited at various scales. Large scale wind farms, involving some hundreds of wind turbines, could supply the electricity grid with a significant proportion of its

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energy needs; one study envisaged up to 25% (Gibbons *et al.*, 1979). Isolated communities, such as on the islands, where electricity costs are particularly high, could be supplied by medium scale turbines. One interesting small scale project is the heating of greenhouses, where the demand for energy correlates highly with its availability. There are many difficult and different problems in this broad task; here we concentrate on the quantification of the resource at a specific site.

Data on the availability of wind energy in Ireland are sparse, because few data exist at locations of interest for wind energy purposes. Only at twelve synoptic meteorological stations have detailed and reliable records of wind speeds and directions been compiled over a long period; see Figure 1. These data are described in section 2. Data at a potential site will typically be few or non-existent, although there may be some broad guidance available from an experienced evaluation of the local topography. The kinetic energy in the wind, and the power available to any specific turbine, is a non-linear function of wind speed, and it is therefore necessary to estimate the full distribution of wind speeds, and not merely an average.

Before starting to operate a wind turbine or wind farm at a potential site, it would be usual to record wind data there for a short period of several weeks or months. As we shall see in section 3, a simple estimator based solely on such a short run of data performs poorly, and a better estimator results by "adjusting" such an estimator, using the long-term records at the synoptic stations. This estimator is calculated by first deseasonalising the data, and then applying a general least squares approach. Given our spatial context, this has close links with "kriging". However, the standard errors yielded by standard assumptions of temporal independence, or even short-memory temporal dependence, are much too small, due mainly to the presence of temporal persistence, namely non-negligible dependence between observations a long time span apart.

We therefore develop alternative interval estimators based on a simple and parsimonious approximating model which accounts for the main features of the data, namely seasonality, spatial correlation, short-memory temporal dependence, and temporal persistence; see section 4.

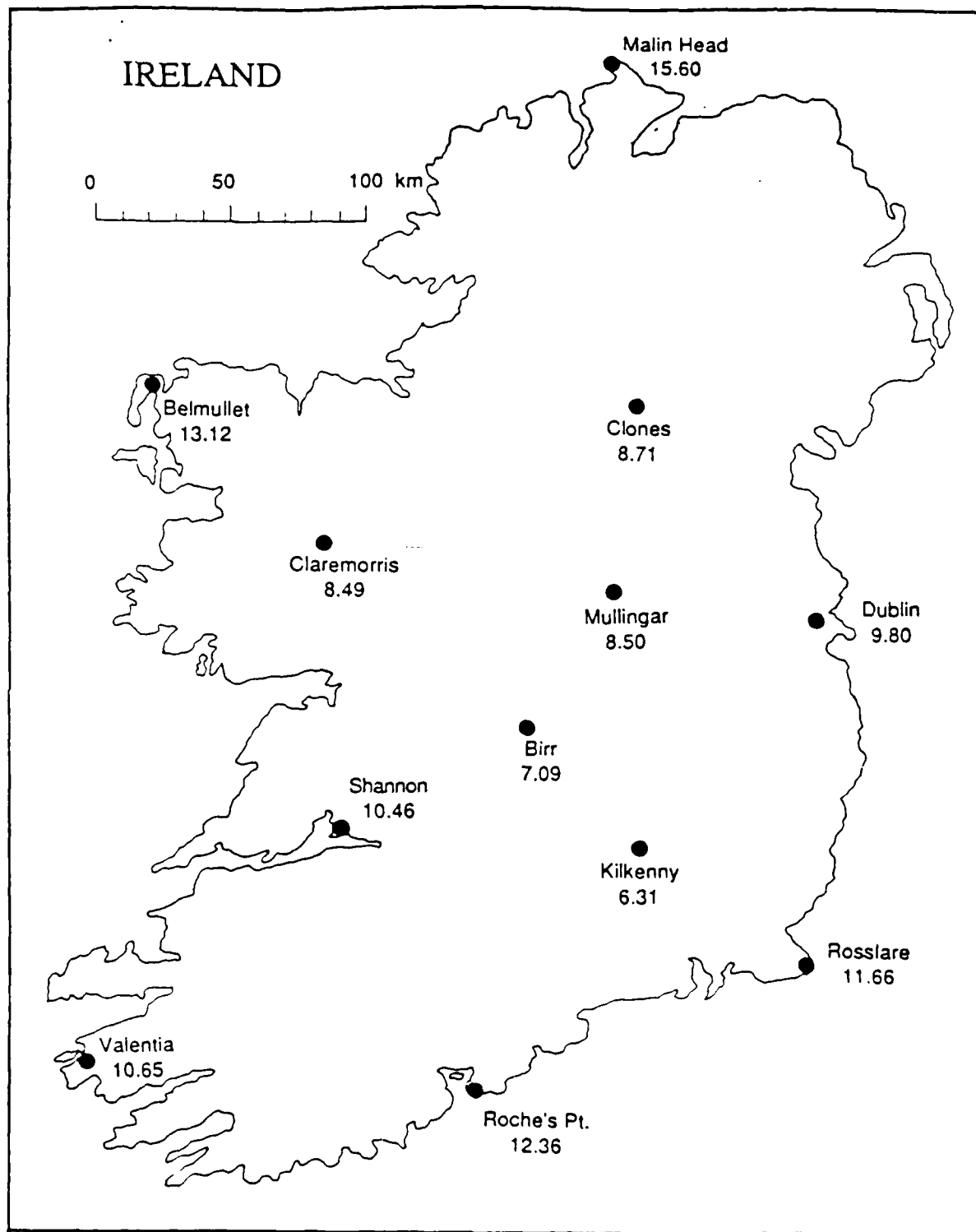


Fig. 1. Irish Meteorological Service synoptic meteorological stations, with mean wind speeds (in knots) for 1961-1978.



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The resulting standard errors are quite accurate. The model also yields estimates of the full distribution of wind speeds, and hence of the available kinetic energy in the wind; see section 5.

2. THE DATA

The data are hourly wind speeds in knots (1 knot = 0.5148 metres per second), and directions at each of twelve synoptic meteorological stations during the period 1961-1978; see Figure 1. Here we present the main features of the data that turned out to be important for our purpose; further description can be found in Haslett and Kellady (1979) and Raftery *et al.* (1982).

At all levels of temporal aggregation, standard deviations are correlated with means. Also, marginal distributions are noticeably asymmetric. Standard exploratory techniques suggested taking a square root transformation, and this does, indeed, stabilise the variance over both stations and time periods, and make the marginal distributions approximately normal. This same transformation has been found to be helpful in similar contexts by other workers; see Brown *et al.* (1984) and Carlin and Haslett (1982).

Another problem is the choice of level of temporal aggregation. For example, Brown *et al.* (1984) work with hourly data, Barros and Estevans (1983) use weekly averages, and Balling and Cervany (1984) rely on monthly aggregates. Our procedure for estimating power at the new site is in two stages: first, estimating the distribution of wind speeds at the new site, which is easier for more aggregated data, and, second, translating this into available kinetic energy, which is easier for less aggregated data. The extent to which speeds at one station can be predicted from contemporaneous speeds at others increases quite rapidly up to a level of aggregation of about one day, and thereafter more slowly. Guided by these observations, we have based our analyses on the square roots of daily mean wind speeds.

Wind speeds vary with time of year, although the seasonal effect is not very strong, accounting for about one-quarter of the total variance. We estimated the seasonal effect by calculating the average of the square roots of the daily means over all years and stations for each

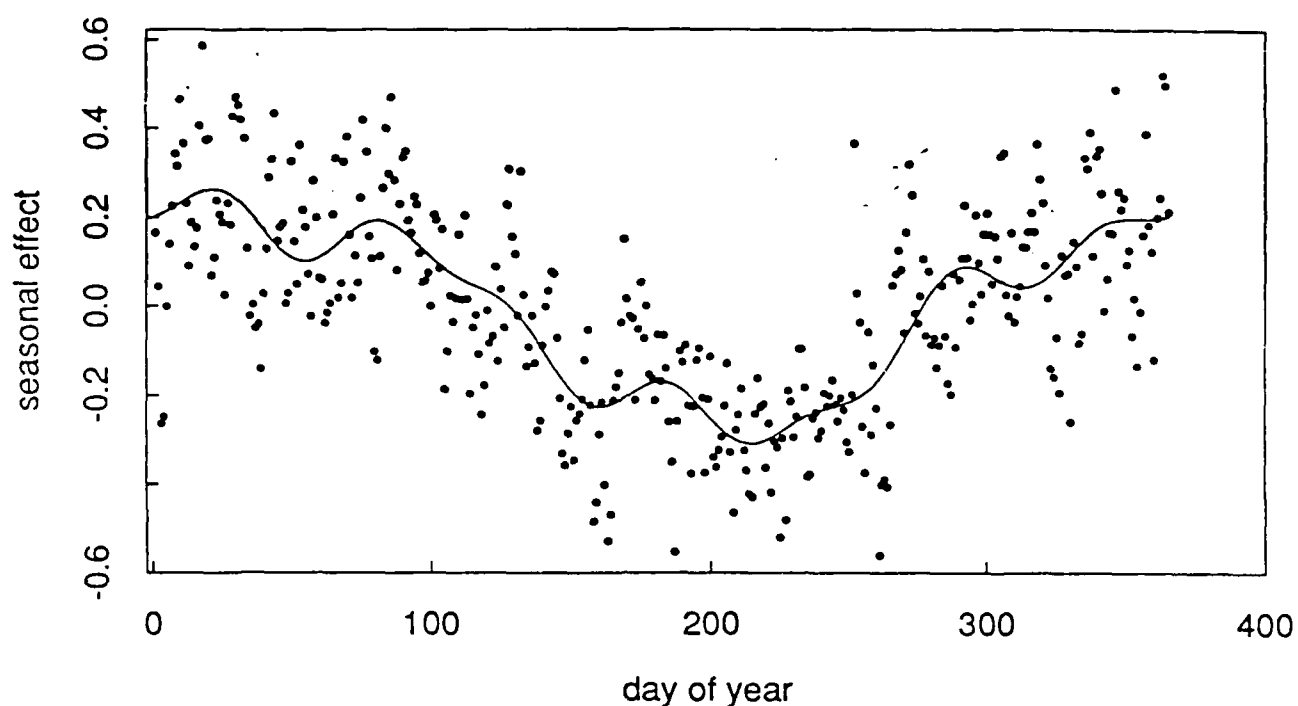


Fig. 2. Seasonal effects. The dots show the average of the square root of the daily means over all stations and years, for each day of the year. The solid line is the estimated seasonal effect.

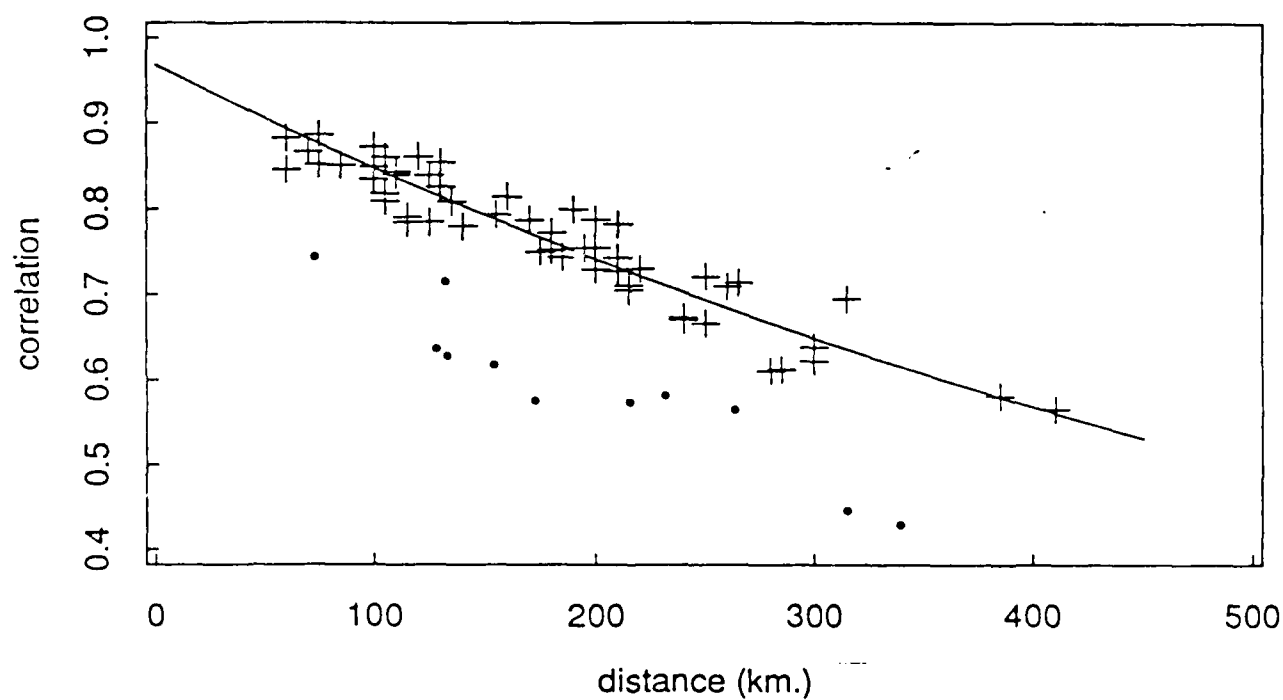


Fig. 3. Distance-correlation plot. Each cross corresponds to a pair of synoptic stations; the dots correspond to pairs which include Rosslare. The solid line is the fitted relationship (3.3) with $\alpha = .968$ and $\beta = .00134$.

day of the year, and then regressing the results on a set of annual harmonics; see Figure 2. Subtraction of the estimated seasonal effect from the square roots of the daily means then yields deseasonalised data, hereafter referred to as velocity measures.

Contemporaneous velocity measures at different stations are highly correlated, and the correlations are clearly related to the distances between stations; see Figure 3. The correlations involving one station - Rosslare - are much lower than the others. This may be because its position in the extreme south-east of the country makes it subject to meteorological influences which do not affect the other stations. We have omitted Rosslare from our calculations. The spatial correlation pattern revealed by Figure 3 changes little with time of year.

The data exhibit some short-memory temporal autocorrelation; see Figure 4. There are striking similarities between its pattern and extent at the different stations.

3. A KRIGING ESTIMATOR

Suppose we have data on wind velocity measures at m places, labelled $1, \dots, m$. One of these, the new site, is labelled k , and there we have a short run of data collected on n consecutive days, $t = t_0, \dots, t_0 + n - 1$. At the other places, the synoptic meteorological stations, we have long runs of data collected on N consecutive days, $t = 1, \dots, N$. Here $n \ll N$, and $1 \leq t_0 < t_0 + n - 1 \leq N$. Let $X_t = (X_{1t}, \dots, X_{mt})^T$, where X_{it} is the velocity measure at place i on day t . Then, as we shall see in section 5, knowledge of the mean and variance of X_k can be translated into quite precise knowledge of the average available kinetic energy in the wind at the new site. This section and the next one are mainly concerned with estimating the mean and variance of X_k .

Let $\mu_i = E[X_{it}]$ ($i = 1, \dots, m$), and $\bar{X}_{i,t_0,n} = n^{-1} \sum_{s=1}^n X_{i,t_0+s-1}$. A simple estimator of μ_k is $\bar{\mu}_k = \bar{X}_{k,t_0,n}$, the average of the observed velocity measures at place k . A simple estimator of the variance of $\bar{\mu}_k$ is

$$\bar{\sigma}_{k,t_0,n}^2 = \{n^{-1} \sum_{s=1}^n (X_{k,t_0+s-1} - \bar{\mu}_k)^2\} / (n-1) \quad (3.1)$$

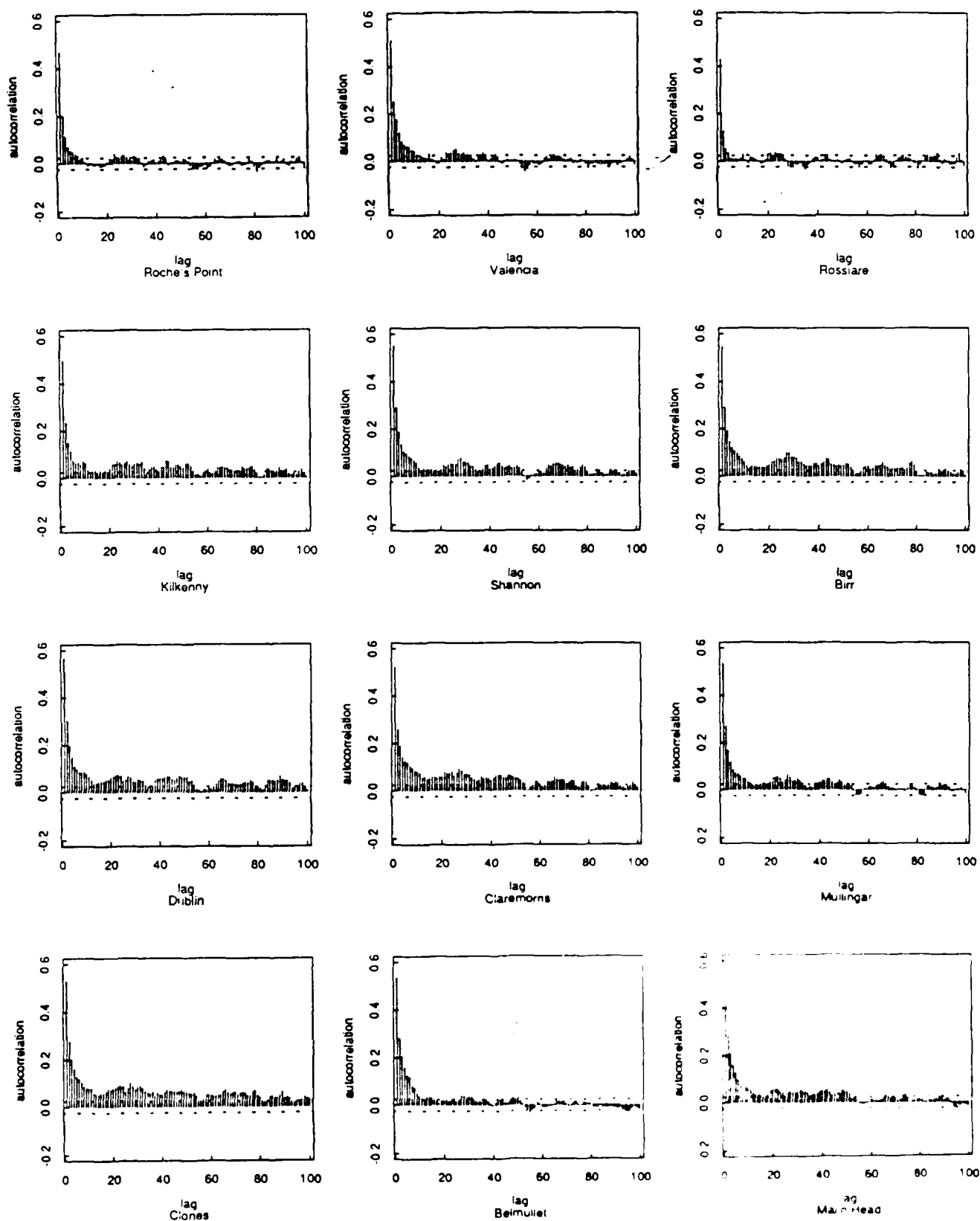


Fig. 4. Autocorrelation functions of the velocity measures for the 12 synoptic stations.

$\bar{\mu}_k$ takes no account of the long runs of data at other places, and we now develop an estimator which does exploit these. The analyses in section 2 indicate that the correlation between wind speeds at different places is strongly related to the distance between them, and suggest that the covariance structure can be reasonably well approximated by the relations

$$\text{Cov}(X_{it}, X_{jt}) = \sigma_X^2 r_{ij} \quad (3.2)$$

where

$$r_{ij} = \begin{cases} 1 & \text{if } i=j \\ \alpha \exp(-\beta d_{ij}) & \text{if } i \neq j \end{cases} \quad (3.3)$$

In (3.3), $0 \leq \alpha \leq 1$, $\beta \geq 0$, and d_{ij} is the distance (in kilometres) between places i and j . If $\alpha < 1$ there is a "nugget effect" due to measurement error and very small-scale effects (Journel and Huijbregts, 1978).

If one assumes that (3.2) and (3.3) hold, and ignore temporal dependence, the general least squares estimator of μ_k is

$$\tilde{\mu}_k = a_k^T (\bar{X}_{t_0,n} - H^{(k)} \ddot{\mu}) a_{kk} \quad (3.4)$$

In (3.4), $A = R^{-1}$, where A is the $m \times m$ matrix with elements (a_{ij}) , and R is the $m \times m$ matrix with elements (r_{ij}) defined in (3.3). Also, $a_k = (a_{1k}, \dots, a_{mk})^T$; $\bar{X}_{t_0,n} = (\bar{X}_{1,t_0,n}, \dots, \bar{X}_{m,t_0,n})^T$; $\ddot{\mu} = (\ddot{\mu}_1, \dots, \ddot{\mu}_m)^T$, where

$$\ddot{\mu}_i = \begin{cases} N^{-1} \bar{X}_{1,1,N} & (i \neq k) \\ 0 & (i = k) \end{cases}$$

and $H^{(k)}$ is the $m \times m$ matrix with elements

$$h_{ij}^{(k)} = \begin{cases} 1 & \text{if } i=j \text{ and } i \neq k \\ 0 & \text{if not} \end{cases}$$

(3.4) was obtained by first approximating the spatial covariance structure by a smooth, isotropic, function of distance, and then using the resulting general least squares estimator. This approach underlies many "kriging" procedures (Journel and Huijbregts, 1978; Ripley, 1981), and

we therefore call $\tilde{\mu}_k$ a kriging estimator. It has also been used in meteorological applications, where it is known as "optimal" or "objective interpolation" (Gandin, 1965; Creutin and Obled, 1982; Tabios and Salas, 1985; Thiebaut and Peddar, 1986). The results are fairly robust to some misspecification of the spatial covariance structure (Cressie, 1985; Brooker, 1986).

It is readily verified that, if (3.2) holds, then $\tilde{\mu}_k$ is unbiased. If we assume that the observations are temporally uncorrelated, then

$$\text{Var}(\tilde{\mu}_k) = \sigma_X^2 a_k^T R a_k / a_{kk}^2 n \quad (3.5)$$

If one assumes, in addition, that $a_k^T X_t$ is normally distributed, which does seem to be approximately the case for our data, then $\tilde{\mu}_k$ is also normally distributed, and interval estimators result from (3.5). Even if $a_k^T X_t$ is not normally distributed, $\tilde{\mu}_k$ will still be approximately normally distributed in large samples, under regularity conditions.

In order to assess the performance of $\tilde{\mu}_k$ as compared with the simpler estimator $\bar{\mu}_k$, and also to assess the variance estimators given by (3.1) and (3.4), we carried out an approximate cross-validation exercise, the results of which are shown in Table 1. For each of several values of n we calculated $\tilde{\mu}_k$ and $\bar{\mu}_k$ for each disjoint data run of length n at each station in turn, and compared these with $\ddot{\mu}_k$, considered to be the "true" value for this purpose. α , β , and σ_X^2 were estimated once from the entire data set, as described in section 4; this provides a good approximation to a complete recomputation on the deletion of each station in turn.

$\tilde{\mu}_k$ performs better than $\bar{\mu}_k$, particularly for short data runs. For example, for runs of length $n = 20$ days, using $\tilde{\mu}_k$ rather than $\bar{\mu}_k$ reduces the empirical mean squared error by about 68%. It appears, empirically, that to achieve the same precision, the simpler estimator would require about six times as much data. The gain in precision decreases as n increases. The empirical distributions of $\bar{\mu}_k$ and $\tilde{\mu}_k$ did not deviate appreciably from normality.

The estimated variances of $\bar{\mu}_k$ obtained from (3.1), and those of $\tilde{\mu}_k$ obtained from (3.5) are, however, clearly quite inaccurate. Not only are they much too small for all values of n , but the extent to which they fall below the empirical variances increases with n . When $n = 320$, they are

TABLE 1
Empirical and theoretical mean squared errors
(multiplied by 10,000) of $\bar{\mu}_k$ and $\tilde{\mu}_k$.

n	MSE ($\bar{\mu}_k$)		MSE($\tilde{\mu}_k$)		
	Empirical	From (3.1)	Empirical	From (3.5)	From (4.10)
20	1156	383	370	129	396
40	710	162	308	64	311
80	493	83	255	32	245
160	302	40	208	16	193
320	197	20	159	8	152

too small by factors of about 10 and 20, respectively. This indicates that simply taking into account the short-memory temporal autocorrelation suggested by Figure 4 would not, by itself, be sufficient to make the standard errors accurate, as it would, effectively, just multiply all the standard errors by a constant.

This suggests that the data exhibit persistence, or long-memory temporal dependence, one of whose manifestations is that the sampling variance of the sample mean decreases more slowly, asymptotically, than the usual rate for short-memory processes, $O(n^{-1})$ (Hosking, 1982, 1984b). Further evidence for persistence is provided by Figure 5, which shows, for each station, the periodogram of the residuals from a fitted AR(9) model. In Figure 5, the short-memory temporal dependence has been largely removed, and yet there is a concentration of power at low frequencies, which is characteristic of long-memory temporal dependence (Graf *et al.*, 1984). The patterns at the different stations are quite similar to each other.

Most models for long-memory temporal dependence imply an approximately linear relationship between power and log-frequency at low frequencies. Inspection of Figure 5 reveals that for our data, at the very lowest frequencies, power is slightly less than such models would lead us to expect. This may be due to the fact that Figure 5 shows the periodogram from a finite sample and not the true spectrum, rather than to inappropriateness of such models. The implied

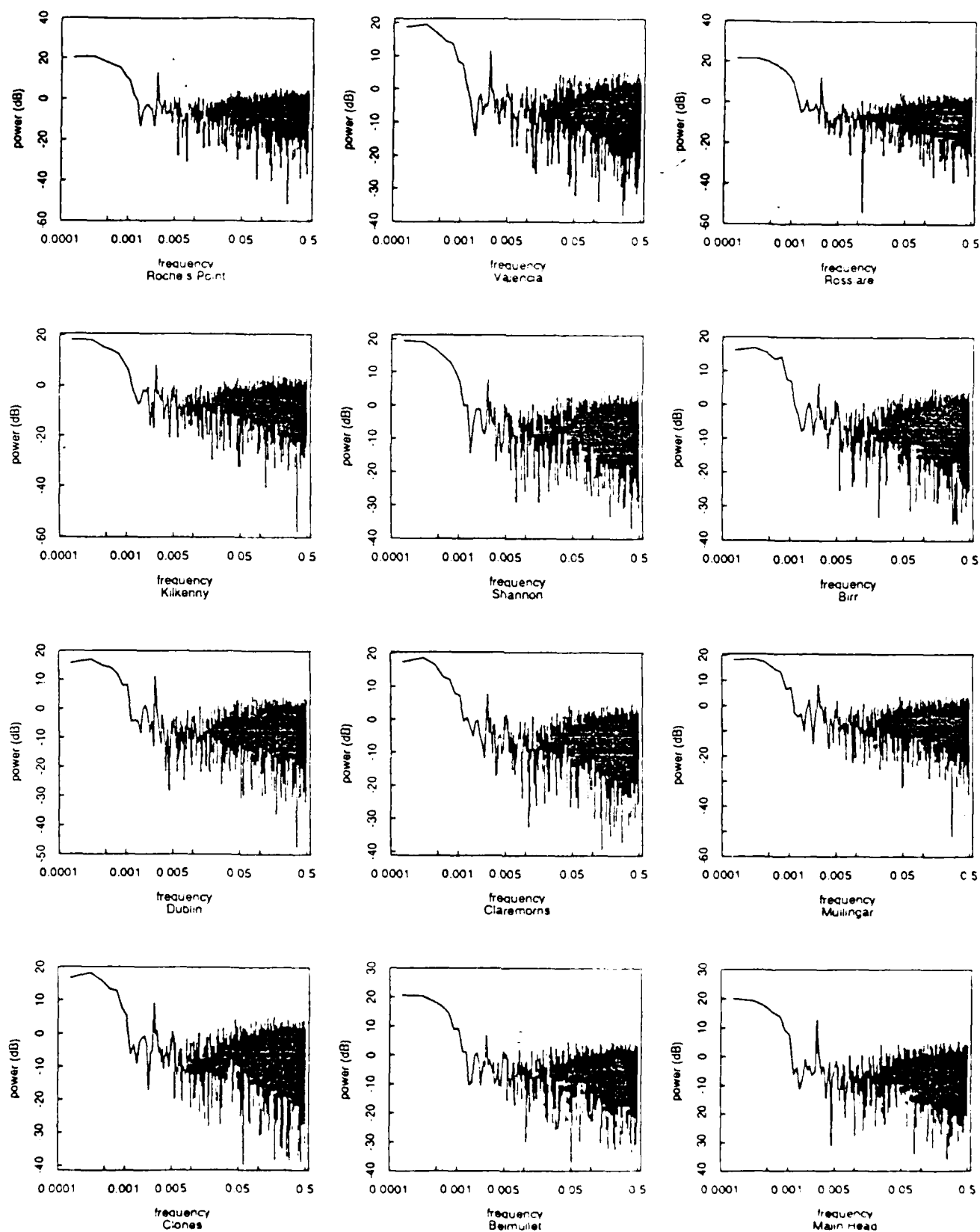


Fig. 5. Spectra of the velocity measures for the 12 synoptic stations, after being passed through a short-memory AR(9) filter.

truncation of the autocovariance function may lead to some negative bias at the lowest frequencies.

4. MODELLING AND INTERVAL ESTIMATION

4.1. A model

We base inference for μ_k on a single model for the entire space-time process $\{X_t : t \in \mathbb{Z}\}$, as follows.

$$X_{it} = \mu_i + \nabla^{-d} \phi(B)^{-1} \theta(B) \epsilon_{it} \quad (4.1)$$

where

$$\epsilon_t = (\epsilon_{1t}, \dots, \epsilon_{mt})^T \stackrel{\text{iid}}{\sim} \text{MVN}(0, \sigma_\epsilon^2 R) \quad (4.2)$$

In (4.1), B is the backward shift operator such that $B \epsilon_{it} = \epsilon_{i,t-1}$; $\nabla = (1-B)$; $\phi(B) = (1 - \phi_1 B - \dots - \phi_p B^p)$; and $\theta(B) = (1 - \theta_1 B - \dots - \theta_q B^q)$. It is assumed that $0 \leq d \leq 1/2$ and that the zeros of $\phi(B)$ and $\theta(B)$ lie outside the unit circle. ∇^{-d} is defined by the binomial series expansion of $(1-B)^{-d}$.

In (4.1), temporal persistence is modelled by the use of fractional differencing (Granger and Joyeux, 1980; Hosking, 1981). (4.1) implies that the second-order moment structure of the space-time process is isotropic and stationary in both space and time. This is true approximately, but not exactly; assuming it enables the resulting model to be applied easily at a new site for which little other information about spatial and temporal covariances is available.

4.2. Identification and preliminary estimation

We used the following procedure to identify the orders of the polynomials $\phi(B)$ and $\theta(B)$, and to obtain initial estimates of the parameters. This is important, because the more exact estimation method described in section 4.3 is feasible only if reasonable starting values are

available. The full data set, consisting of $N = 6574$ observations at each of $m = 11$ stations, was used.

1. Form preliminary estimates of α and β by regressing $\log\{\text{Corr}(X_{it}, X_{jt})\}$ on $\log(d_{ij})$, using the fact that $\text{Corr}(X_{it}, X_{jt}) = \text{Corr}(\epsilon_{it}, \epsilon_{jt})$ for (4.1).
2. Form m approximately spatially independent series $\{Y_{it} : t = 1, \dots, N\}$ ($i = 1, \dots, m$), where $Y_t = CX_t$, and $Y_t = (Y_{1t}, \dots, Y_{mt})^T$. Here C is a lower triangular $m \times m$ matrix such that $CRC^T = I$; C is constructed by the Gram-Schmidt orthogonalisation procedure.
3. Find an autoregressive filter which accounts for most of the short-memory dependence in the $\{Y_t\}$ series, and filter each of the m series $\{Y_{it} : t = 1, \dots, N\}$ with it. This yields a data set which is free, to a good approximation, of both spatial and short-memory temporal dependence, and whose main feature is persistence. The filter we used was of order nine.
4. Form means of n consecutive values from the output of step 3, for several subsets of the data, and several values of n . For each value of n , calculate the sampling variance of these sample means. Then, regress the logarithm of the sampling variance on $\log n$, and take one-half of one minus the estimated slope as an initial estimate of d , \bar{d} , say. This is motivated by the asymptotic results of Hosking (1982, 1984b).
5. Form the m series $\{\nabla^{\bar{d}} Y_{it} : t = 1, \dots, N\}$. A fast and accurate way of implementing this filter is described in section 4.3.
6. Identify a common ARMA (p, q) model for the m series $\{\nabla^{\bar{d}} Y_{it} : t = 1, \dots, N\}$, and estimate it, yielding initial estimates of $\phi(B)$ and $\theta(B)$. We identified an AR(2) model for our data.

4.3. Maximum likelihood estimation

Let $X^t = \{X_1, \dots, X_t\}$ and $X_i^t = \{X_{i1}, \dots, X_{it}\}$. Then the likelihood can be calculated exactly by noting that, conditionally on X^{t-1} , X_t has a multivariate normal distribution, where $E[X_{it} | X^{t-1}] = E[X_{it} | X_i^{t-1}]$, $\text{Var}[X_{it} | X^{t-1}] = \text{Var}[X_{it} | X_i^{t-1}]$, and $\text{Corr}(X_{it}, X_{jt} | X^{t-1}) =$

$\alpha \exp(-\beta d_{ij})$. The one-dimensional conditional means, $E[X_{it} | X_i^{t-1}]$, and variances, $\text{Var}[X_{it} | X_i^{t-1}]$, may be calculated by inserting the autocorrelations for the fractionally differenced ARIMA (p, d, q) process (Hosking, 1981) into the Durbin-Levinson recursion (Ramsey, 1974).

Maximum likelihood estimators can then be found by numerically maximising the likelihood. This is, however, a demanding task, computationally. For example, a single evaluation of the likelihood takes about three hours of CPU time on a VAX 11/780, and finding the maximum likelihood estimator would take at least 45 hours, even with good starting values. Carlin *et al.* (1985) and Carlin (1987) did obtain estimates for long-memory time series models by numerically maximizing the exact likelihood, which seems to have been practicable because they were working with short, one-dimensional, series of lengths less than 220.

However, a fast and accurate approximation can be found, as follows. First, we note that, to an excellent approximation, the conditional means and variances may be found using only the partial autocorrelations for the fractionally differenced ARIMA $(0, d, 0)$ process, and not those for the full ARIMA (p, d, q) process, which are much more complicated (Hosking, 1981). Then we can find the maximum likelihood estimators of μ and σ_ϵ^2 analytically, and find a concentrated likelihood which is a function only of α , β , d , $\phi(B)$, and $\theta(B)$. Finally, we approximate the partial linear regression coefficients of the ARIMA $(0, d, 0)$ process, the calculation of which dominates the CPU requirements. The resulting approximation reduces the required CPU time by a factor of about 70 for our data, and appears to be quite accurate.

We now describe the approximation used in more detail. We note, first, that to an excellent approximation,

$$E[X_{it} | X_i^{t-1}] = u_{it} + w_t \mu_i \quad (4.3)$$

and

$$\text{Var}[X_{it} | X_i^{t-1}] = \sigma_X^2 \kappa \prod_{j=1}^{t-1} (1 - \phi_{jj}^2) = v_t \quad (4.4)$$

In (4.3) and (4.4)

$$u_{it} = \phi(B)\theta(B)^{-1} \sum_{j=1}^{t-1} \phi_{ij} X_{i,t-j} \quad (4.5)$$

$$w_t = 1 - \phi(1)\theta(1)^{-1} \sum_{j=1}^{t-1} \phi_{ij} \quad (4.6)$$

where the ϕ_{ij} are the partial linear regression coefficients for the ARIMA (0,d,0) process, given explicitly by Hosking (1981), and κ is the ratio of the innovations variance to the process variance for the ARMA (p,q) process with parameters $\phi(B)$ and $\theta(B)$, as defined by equation (3.4.4) of Box and Jenkins (1976).

Given values of α , β , d , $\phi(B)$, and $\theta(B)$, the maximum likelihood estimators of μ and σ_ϵ^2 are then available analytically as

$$\hat{\mu}_i = \sum_{t=1}^N w_t (X_{it} - u_{it}) v_t^{-1/2} / \sum_{t=1}^N w_t^2$$

$$\hat{\sigma}_\epsilon^2 = (Nm)^{-1} \sum_{t=1}^N (X_t - u_t - w_t \hat{\mu})^T A (X_t - u_t - w_t \hat{\mu}) v_t^{-1}$$

where $u_t = (u_{1t}, \dots, u_{mt})^T$ and $\hat{\mu} = (\hat{\mu}_1, \dots, \hat{\mu}_m)^T$. The concentrated log-likelihood is then

$$l(\alpha, \beta, d, \phi(B), \theta(B)) = \text{constant} - 1/2 Nm \log(\sigma_\epsilon^2) - 1/2 N |R| \quad (4.7)$$

and maximum likelihood estimators can be found by maximising this numerically. It is computationally efficient to maximise l as a function of the other variables conditionally on d , for each trial value of d .

The recursive calculation of the ϕ_{ij} and the evaluation of (4.3) dominate the CPU time requirements. We approximate these, noting that by Hosking (1981), $\phi_{ij} \sim -\pi_j$, and $\pi_j \sim O(j^{-d-1})$ as $t \rightarrow \infty$, where the π_j are the π -weights of the ARIMA (0,d,0) process, as defined by Box and Jenkins (1976). Our approximation consists of taking these asymptotic relationships to hold exactly for $j > M$, where M is some integer, and then taking the π_j to be constant, at their approximate average value, for $M < j \leq t-1$. This yields

$$\begin{aligned} \sum_{j=1}^{t-1} \phi_{tj} X_{i,t-j} &\approx \sum_{j=1}^M \phi_{tj} X_{i,t-j} + \sum_{j=M+1}^{t-1} \pi_j X_{i,t-j} \\ &\approx \sum_{j=1}^M \phi_{tj} X_{i,t-j} + M \pi_M d^{-1} \{1 - (M/j)^d\} \bar{X}_{i,M+1,t-1-M} \end{aligned} \quad (4.8)$$

(4.8) is then substituted into (4.5), and a similar approximation is used in (4.6). This provides a good approximation to the likelihood function as a whole, and so opens the possibility of Bayesian, as well as likelihood, inference for fractionally differenced models. Numerical experimentation indicated that choosing $M=100$ gives good results over a wide range of values of d and N . For our application, the approximation reduced CPU time by a factor of about 70, so that the CPU time required for a single evaluation of the likelihood went down from three hours to about 2½ minutes.

Hosking (1984a) proposed an approximation which is similar in spirit to (4.8). However, (4.8) seems to be somewhat faster for the very long series we are dealing with here ($N=6,574$), and seems also to avoid the starting value problem for the long-memory filter. Spectral approximations to the maximum likelihood estimator in the one-dimensional case have been proposed by Fox and Taquq (1986), Beran (1986), and Geweke and Hudak-Porter (1983); these have not, however, been generalised to the multivariate, or space-time, context. Short-memory autoregressive approximations have been suggested by Granger and Joyeux (1980) and Li and McLeod (1986), but we felt it important to retain the long-memory property in the approximation used.

Neither the finite sample, nor the asymptotic distribution of the maximum likelihood estimator for models such as (4.1) appears to be known. However, Yajima (1985) has shown that the maximum likelihood estimator for a one-dimensional specialisation of (4.1), namely the fractionally differenced ARIMA (0, d ,0) process with known mean, has the usual asymptotic normal distribution, while a similar result for the purely spatial specialisation of (4.1) follows from Mardia and Marshall (1984). We conjecture that the usual result does hold for (4.1), and could be proved by combining the arguments used in these two papers.

The approximate maximum likelihood estimators, with their approximate standard errors in parentheses, are $\hat{\alpha} = .968 (.0013)$, $\hat{\beta} = .00134 (.000025)$, $\hat{d} = .328 (.0029)$, $\hat{\phi}_1 = .010 (.0123)$, $\hat{\phi}_2 = -.063 (.0123)$, and $\hat{\sigma}_\epsilon^2 = .477$. Thus, the long-memory effect is found to be both large and significant, and there is a fairly small, but significant, nugget effect, indicated by $\hat{\alpha}$ being significantly less than one.

4.4. Model checking

Suppose $e_t = C (X_t - u_t - w_t \hat{\mu}) v_t^{-1/2} = (e_{1t}, \dots, e_{mt})^T$. Then, conditionally on the estimated model, the e_{it} should be independent and identically distributed normal random variables with mean zero, and variance σ_ϵ^2 . The quantile-quantile plots, autocorrelations, and cumulative periodograms of the residuals are in good agreement with this. There is, however, a small number of clearly non-zero cross-correlations at lag zero. These are all less than 0.2 in absolute value, and reflect the fact that, as can be seen from Figure 3, the assumed spatial correlation function (3.3) is not exact; this seems unavoidable. The most important check on the model for our purposes is the quality of the interval estimates it yields; this is investigated in section 4.5.

In analysing another, one-dimensional, set of wind speed data, Lawrance and Lewis (1985) concluded, using techniques based on third-order moments, that their data exhibited non-linearity. Similar analyses, and also calculations based on fourth-order moments (Granger and Andersen, 1978), failed to reveal any such non-linearities in our data.

4.5. Interval estimation at a new site

Let us now consider the situation of section 3, where we have a short run of data at a new site k , on the basis of which we wish to estimate μ_k . As we saw, the estimator $\bar{\mu}_k$ defined by (3.4) performs well as a point estimator. An interval estimator now follows by noting that, conditional on the model (4.1), $\bar{\mu}_k$ is unbiased and normally distributed with

$$\text{Var}(\bar{\mu}_k) = \sigma_X^2 a_k^T R a_k \{n + \sum_{j=1}^{n-1} (n-j) \rho_j\} / n^2 \quad (4.9)$$

where ρ_j are the autocorrelations for the fractionally differenced ARIMA (p, d, q) process. However, (4.9) is inconvenient because the ρ_j are rather complex. A convenient approximation to (4.9) is

$$\text{Var}(\tilde{\mu}_k) \approx 2\pi f(0) a_k^T R a_k \left\{ n + \sum_{j=1}^{n-1} (n-j) \rho_j^{(d)} \right\} / n^2 \quad (4.10)$$

where the $\rho_j^{(d)}$ are the autocorrelations of an ARIMA $(0, d, 0)$ process given explicitly by Hosking (1981), and $f(0)$ is the spectrum at zero of the estimated ARMA (p, q) model with parameters $\hat{\phi}(B)$ and $\hat{\theta}(B)$, and innovations variance σ_ϵ^2 (Fuller, 1976, Theorem 6.1.2).

To assess the accuracy of standard errors based on (4.10), we extended the cross-validation exercise reported in section 3 to include them; see Table 1. The theoretical standard errors are quite accurate, and, in particular, they capture the non-standard rate of decline of the mean squared error of the empirical errors quite well.

It would, of course, be possible to use as a point estimator the exact maximum likelihood estimator of μ_k based on the model (4.1), rather than the kriging estimator $\tilde{\mu}_k$. However, this is a much more complicated solution, and some numerical work indicated that there is little to be gained from adopting it. This is not surprising, given the result of Adenstedt (1974) and Beran and Kuensch (1985), that the loss of efficiency incurred by using the sample mean is small for one-dimensional long-memory processes.

5. ESTIMATING WIND POWER

The power P (in W/m^2) in a wind with instantaneous speed V (in knots) is due to its kinetic energy, and is

$$P = \frac{1}{2} \rho V^3 \quad (5.1)$$

where ρ depends on the density of air, and is, in these units, $\rho = .167$ (Golding, 1955). Not all of this energy is available to a given turbine. Indeed, there is a theoretical upper bound of 0.593 (the Betz coefficient) to the proportion of this energy which may be extracted. The amount

which is available to a particular turbine is a complicated function of V , the parameters of which are specific to that turbine. A realistic figure for a modern efficient turbine is on the order of 0.35. For maximum generality, therefore, we concentrate here on estimation of the long-run average of P at a new site. Brown *et al.* (1984) give an example of the computation of windpower from a specific turbine.

By (5.1), the average power in the wind at place i on day t is $1/2\rho\overline{V_u^3}$, where $\overline{V_u^3}$ is the average cubed wind speed at place i on day t . However, we have formulated our model in terms of the velocity measures X_u . In order to deduce an empirical relation between $\overline{V_u^3}$ and X_u we constructed a log-log plot of $\overline{V_u^3}$ against $Z_{it}=X_{it}+s_t$, where s_t is the seasonal effect for day t . This is shown in Figure 6, and is based on 180 days sampled randomly from each of the synoptic meteorological stations.

Figure 6 suggests the approximate relation

$$E[\overline{V_u^3}|Z_t] = \gamma Z_{it}^\delta \quad (5.2)$$

with parameters which are approximately constant across place and time of year. We have taken $\delta=5$, which accounts for almost as much of the variation (97%) as the best fitting line (for which $\delta=4.6$), and yields the very simple result

$$\begin{aligned} E[\overline{V_u^3}] &= \gamma E[Z_{it}^5] \\ &= \gamma \{(\mu_i+s_t)^5 + 10(\mu_i+s_t)^3\sigma_X^2 + 15(\mu_i+s_t)\sigma_X^4\} \end{aligned} \quad (5.3)$$

The estimated value of γ is 5.06.

To obtain an estimate, \hat{P}_k , for the average power at place k , we replace μ_k by $\tilde{\mu}_k$ in (5.3), and average over all values of t for a year. This yields

$$\hat{P}_k = 1/2\rho\gamma \frac{1}{365.25} \sum_{t=1}^{365} \omega_t \{(\tilde{\mu}_k+s_t)^5 + 10(\tilde{\mu}_k+s_t)^3\sigma_X^2 + 15(\tilde{\mu}_k+s_t)\sigma_X^4\} \quad (5.4)$$

where ω_t is 0.25 for 29 February, and 1.0 for all other days. Upper and lower confidence bounds for P_k may be obtained by replacing $\tilde{\mu}_k$ in (5.4) by the upper and lower confidence bounds for

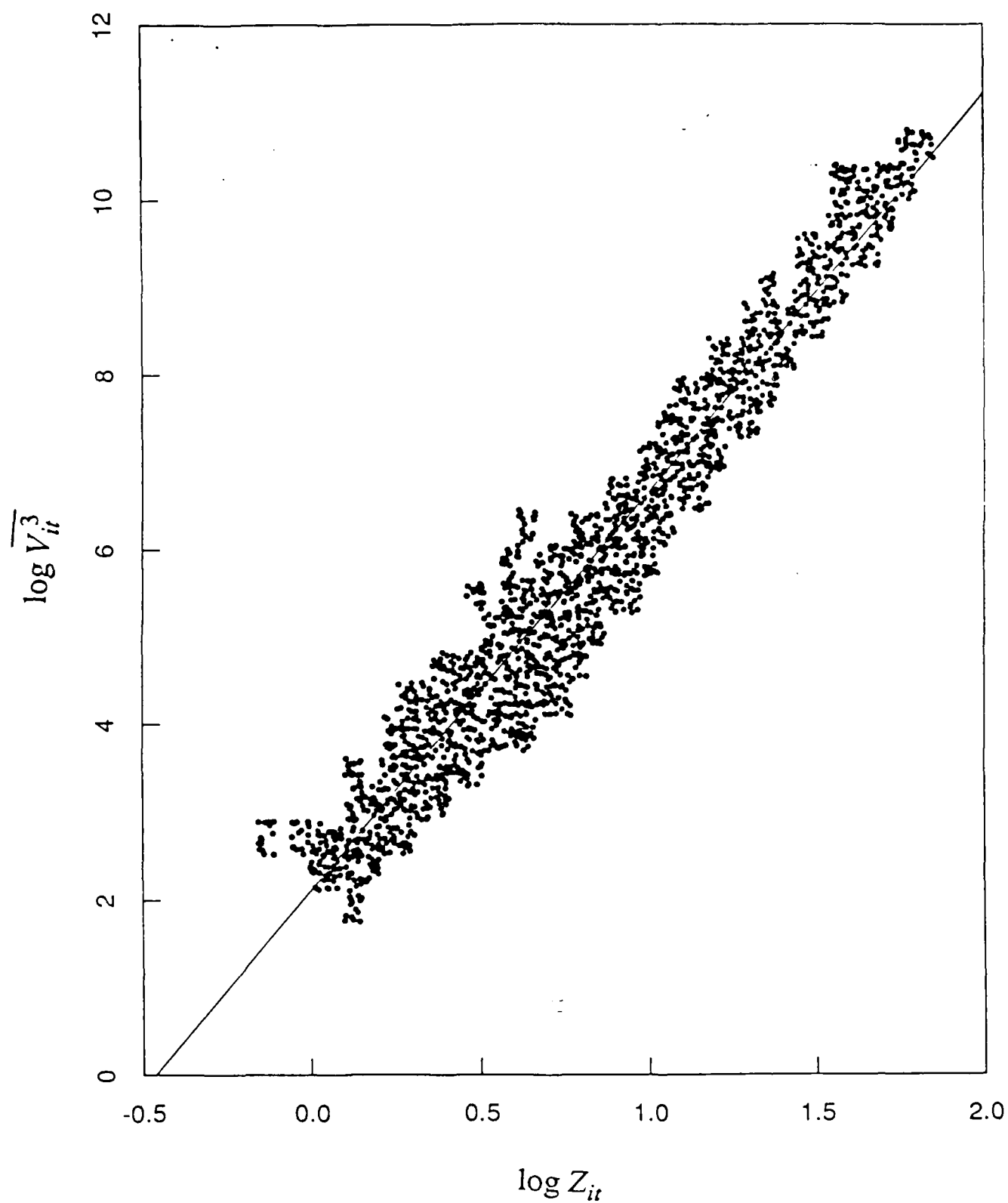


Fig. 6. Plot of log average cubed wind speed ($\log \overline{V_{it}^3}$) against log square root of daily average ($\log Z_{it}$) for 180 days sampled randomly from each of the synoptic stations.

μ_k derived in section 4.

In Table 2, we show the results of implementing this procedure for several examples involving each of the synoptic stations, a collection of starting values ranging over years and time of year, and several values of n . The "true" values are obtained from (5.4) with $\bar{\mu}_k$ replaced by $\ddot{\mu}_k$.

TABLE 2
Estimated kinetic energy (in kJ/m^2) based on short data runs at the synoptic stations. The point estimates and confidence intervals are based on (5.4), (3.4), and (4.10).

Site	Starting date	n	Point estimate	"True" value	95% confidence limits	
					lower	upper
Malin Head	1 January, 1961	20	.37	.57	.19	.65
Roche's Point	5 February, 1962	20	.38	.35	.23	.62
Valencia	12 March, 1963	20	.21	.25	.11	.38
Kilkenny	15 April, 1964	40	.10	.09	.06	.16
Shannon	20 May, 1965	40	.24	.24	.16	.35
Birr	24 June, 1966	40	.10	.11	.06	.14
Dublin	29 July, 1967	80	.21	.21	.14	.31
Claremorris	1 September, 1968	80	.18	.15	.12	.26
Mullingar	6 October, 1969	160	.17	.16	.13	.23
Clones	29 January, 1971	160	.14	.16	.10	.20
Belmullet	8 April, 1973	320	.37	.39	.27	.51
Malin Head	22 February, 1974	320	.70	.57	.49	.96

These results reflect the satisfactory performance of the interval estimator of μ_k . They also show how much harder it is to estimate mean kinetic energy than mean speed. For example, in the example of the first line of Table 2, it is possible to estimate μ_k to within about $\pm 13\%$ at a 95% confidence level after 20 days, but mean kinetic energy can be estimated only to within a factor of about 2. The last line of Table 2 illustrates the fact that even when n is greatly

increased, the uncertainty remains considerable.

To bring these results into more concrete form, we note that an average power of 544 W/m^2 (as at Malin Head) corresponds to $4765 \text{ kWh/m}^2/\text{annum}$ of energy. Thus, for a horizontal axis turbine with a 5 m blade, and a cross-section therefore of 79 m^2 , and an average efficiency of 0.35 , this corresponds to an energy production of about 131 MWh/annum on average. For comparison Irish electricity production in 1985-86 was about 10^7 MWh/annum (Electricity Supply Board, 1986).

6. DISCUSSION

We have proposed a procedure for estimating wind power at a new site which gives reasonable answers and is easy to apply in practice. Inference is based on a simple and parsimonious approximating model which synthesises deseasonalisation, kriging, ARMA modelling, and fractional differencing in a natural way.

We have focussed here on the evaluation of the average power output to be expected in the long term from a wind turbine at a given site. We have ignored many questions however. For example, the basic data pertain to a standard height of 10 metres ; the question of height extrapolation is discussed, with references, in Brown *et al.* (1984). Furthermore, other factors, such as variability, influence the value of the wind power resource to, for example, an electricity utility. Short term predictions of power available and required are of considerable importance for the control of an electricity grid. Such questions need model predictions at short time scales, on the order of one hour or less, and are not provided by this model; see Brown *et al.* (1984) and Lou and Corotis (1985). The long term variability of the resource is another key issue in discussions of questions such as the "capacity credit" of a proposed wind farm; see Haslett and Diesendorf (1981) and Carlin and Haslett (1982). The model developed in section 4 provides some basis for answering this, and other, questions about the resource. For example, it could be used for medium-term prediction and control.

Our model is based on the assumption that the second-order moment structure of the space-time process is constant over Ireland (except the south-east corner). This assumption holds approximately, but not exactly, and it plays the important role of providing an estimate of the second-order properties at a new site for which there is not enough data to estimate them independently. Thus, the error bounds above must be somewhat optimistic, being conditional on the estimated values of all the parameters of the model, other than the mean, and on their being site independent. A natural way to relax it would be to cast the problem in a parametric empirical Bayes framework (Morris, 1983). This would be much more complicated, and some numerical work indicated that, in our case, neither exact knowledge of the second-order moment structure, nor the imposition of a joint probability distribution on the μ_i would greatly improve precision over our method. However, it may well be worth investigating for other problems of this type.

We have not made use of the available wind direction information. Wind turbines are able to turn quite rapidly so as to be optimally placed for electricity generation with respect to the current wind direction. Thus information on wind directions would be of use only indirectly in assessing the resource, if it enabled us to estimate the distribution of wind speeds with more precision; our calculations suggest that it would not. For example, we decomposed each wind speed into components parallel and perpendicular to the prevailing wind direction, an approach advocated by McWilliams and Sprevak (1985). The strong relationship between spatial correlation and distance shown in Figure 3, which is crucial to our method, disappeared. Even assuming exact knowledge of all spatial correlations did not lead to appreciable improvement in the estimator of μ_k when wind speeds were decomposed into components.

One refinement which may well lead to increased precision is the incorporation of prior expert opinion about μ_k , usually that of a meteorologist. This can be done using Bayes' theorem. A simple, approximate, Bayesian solution follows by noting that posterior uncertainty about μ_k is much greater than that about any of the other parameters, and approximating the likelihood for μ_k by a normal density, obtained by assuming the other parameters to be known exactly. It can

also be done, approximately, in a non-Bayesian way by regarding the prior mean as another estimator, and using the prior variance to combine the two estimators in the usual way. If the prior distribution is normal, these two approaches should give very nearly the same answer.

A variety of alternative approaches could be taken. For example, Deutsch and Pfeiffer (1981) outline a different approach to space-time modelling which introduces spatial structure by ordering the neighbours of each site, rather than by modelling the spatial covariance structure. Their method seems less applicable to the present problem. Another approach is suggested by the meteorological plausibility of considering wind speeds to be governed by regimes which succeed one another according to a renewal process.

The evaluation of wind power has been considered for other countries, including Argentina (Barros and Estevans, 1983), New Zealand (Cherry, 1980), Denmark, the U.K. (Musgrove, 1987; Halliday, 1984), the USA (Pennell *et al.*, 1980; Justus *et al.*, 1976), and Spain (Adell *et al.*, 1987). Many of these make use of meteorological models of the air-flow away from the Earth's surface. However, such studies have, typically, given little explicit indication of the precision of their estimates. Exceptions include Corotis (1977) and Barros and Estevans (1983).

There have been few studies of the spatial co-variability of wind speeds in this context. Exceptions include Balling (1984), Barros and Estevans (1983), Corotis *et al.* (1977) and Carlin and Haslett (1982); these have, however, adopted very simple descriptions only. The only detailed approach known to us of adjusting short series of wind speeds, by reference to longer series, is that of Barros and Estevans (1983), but this ignored the temporal autocorrelations. Such adjustments are well known in the hydrology literature as "augmentation" procedures; see Vogel and Stedinger (1985) for a recent review. These do not, however, typically model the correlations spatially, and usually ignore even the short term temporal auto-correlation.

One consequence of the long-memory property is that the gain in precision from extending records in time at one site rapidly becomes small. However, the availability of even small amounts of data at nearby places increases precision considerably, due to the strong spatial correlation. For the purpose of estimating the wind resource, it therefore seems worthwhile

collecting windspeed data at a much denser grid of locations, perhaps using simple anemometers. This seems likely to be a more efficient use of resources than recording wind speeds for an extended period at a small number of additional locations. Of course, there are parameters of interest other than long run average power, such as extrema, concerning which we can make no such recommendations.

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